INTRODUCTION TO PROBABILITY MODELS

Lecture 8

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REMINDERS

1. The first homework is due **NOW**
2. The second quiz will be on this **Wednesday(Sep 12)**

BASIC COUNTING RULES

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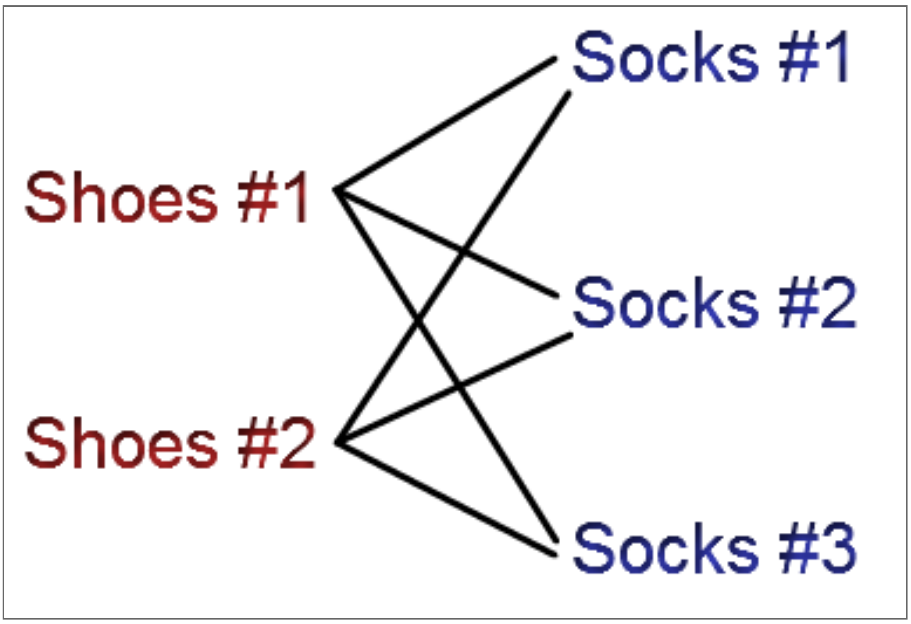
If there are a ways of doing something and b ways of doing another thing, then there are a · b ways of performing both actions.



EXAMPLE 1

Assuming Mary has 6 pairs of shoes, 10 different tops, 8 different bottoms and 4 different jackets.

1. How many different outfits can she wear?
2. Mary has a job interview and she wants to decide what to wear. Of all her clothes, Mary has 2 pairs of shoes, 3 tops, 2 bottoms and 2 jackets that are appropriate for an interview. She randomly picks what to wear for the interview among all her possible outfits, what is the probability that s he wears an interview-appropriate outfit?

EXAMPLE 2

lllinois license plates consist of 4 digits followed by 2 letters. Whereas, in Ohio, license plates start with 3 letters and end with 4 digits. Assume all letters are upper case.(note: the license plate scheme described may not reflect the current Illinois or Ohio license plates)

1. For each state, how many possible license plates are there?
2. How many possible license plates are there for each state with no digit or letter repeating?
3. How many possible license plates are there with at least 1 vowel?
4. How many possible license plates are there with at least one vowel or at least one 3?
5. What is the probability that the license plate will have at least one vowel?

PERMUTATION

TWO CONCEPTS

- Factorial Notation: $k!$ means multiple the positive integer k by $k-1, k-2, \cdots,$ until 1

  $k! = k \times (k-1) \times \cdots \times 2 \times 1$

  $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$
- Permutation: Ordered arrangement of r distinct objects from a set of n objects.

  $${}_nP_r = P_r^n = \frac{n!}{(n-r)!}$$

  $${}_5P_2 = P_2^5 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 20$$

EXAMPLE 3

Suppose Krannert only allows 5 spaces for a password to Portals. Suppose further you are only allowed to use a number or a letter, but the system is not case sensitive.

1. How many possible passwords are there?
2. What is the probability that you do not have a 9 in the first position?
3. What is the probability that all 5 spaces are odd numbers? What if you cannot have a 9 in the first space?
4. What is the probability that a password does not repeat any characters?
5. What is the probability that the first space is a letter?
6. What is the probability that the 4_{th} space is an even number?
7. What is the probability that the last two spaces are vowels, if repeats are allowed? If repeats are not allowed?
8. What is the probability that the password has at least one letter?