INTRODUCTION TO PROBABILITY MODELS

Lecture 6

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INDEPENDENCE

DEFINITION

We say that event B is independent of event A if the occurrence of A does not affect the probability of B occurring

- $P(A|B) = P(A)$
- $P(B|A) = P(B)$
- $P(A \cap B) = P(A) \times P(B)$

ARE MUTUALLY EXCLUSIVE EVENTS ALSO INDEPENDENT?

EXAMPLE 1

Chris and his roommates each have a car. Julia's Mercedes SLK works with probability 0.98, Alex's Toyota Corolla works with probability 0.91, and Chris' GMC Jimmy works with probability 0.24. Assume all cars work independently of one another. What is the probability that at least 1 car works?

EXAMPLE 2

Consider the following sample space $S = \{a1, b1, c1, d1, a2, c2, b3, d3, a4, b4, c4, d4\}$. The probability of each individual outcome is equal. Events are defined as follows:

- E1 contains all elements with a or d
- E2 contains all elements with numeric parts less than 4
- E3 contains all elements with 4 for the numeric part

1. List $E1, E2, E3$ and find $P(E1)$, $P(E2)$, and $P(E3)$
2. List $E1 \cap E3$ and find $P(E1 \cap E3)$.
3. What is $P(E3|E2)$?
4. What is $P((E2 \cup E1)^c)$?
5. Are $E1$ and $E3$ independent? Mathematically justify your conclusion.
6. What term describes the relationship between $E2$ and $E3$? Explain your reasoning? Are $E2$ and $E3$ independent?

MUTUALLY INDEPENDENCE OF THREE SETS

Must statisfy all these conditions

1. $P(A \cap B) = P(A) \times P(B)$
2. $P(A \cap C) = P(A) \times P(C)$
3. $P(B \cap C) = P(B) \times P(C)$
4. $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

We call it pairwise independent if only 1, 2 and 3 hold

EXAMPLE 3

Draw a card out of a standard deck of 52 playing cards. Define the following events: A = the card is a heart, B = the card is a face card, C = the card is a 7 or Jack.

1. Are events A and B independent? Prove your answer mathematically.
2. Are these 3 events pairwise independent?
3. Are they mutually independent?