INTRODUCTION TO PROBABILITY MODELS

Lecture 39

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CALCULATE CORRELATION

- Population
  - Variance: $\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$
  - Covariance: $\sigma_{x,y} = \frac{\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)}{N}$
  - Correlation: $\rho_{x,y} = \frac{\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)}{\sqrt{\sum_{i=1}^{N}(x_i - \mu_x)^2}\sqrt{\sum_{i=1}^{N}(y_i - \mu_y)^2}}$
- Sample
  - Variance: $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$, SD: $s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$
  - Covariance: $s_{x,y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$
  - Correlation: $r_{x,y} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{N}(y_i - \bar{y})^2}}$
  - Sample correlation is often written as $r_{x,y} = \frac{s_{x,y}}{s_x s_y}$

EXAMPLE 1

You want to compare the airspeed velocity of unladen swallows by species. Suppose you collect the following data on pairs of African and European swallows.

African	**European**
18	21
22	22
26	25
30	28

Calculate the means, variances, standard deviations of each. Then calculate the covariance and the correlation between the two species.

LEAST-SQUARES REGRESSION

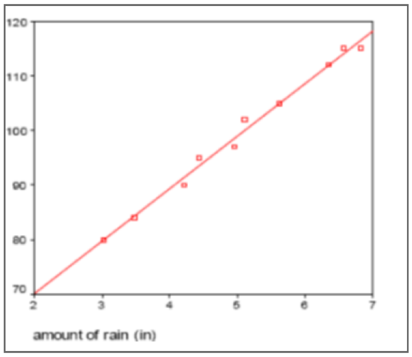
Can we do better than just a scatter plot and the correlation in describing how x and y are related? What if we want to predict y for other values of x? Least-Squares Regression fits a straight line through the data points that will minimize the sum of the vertical distances of the data points from the line

- Minimizes $\sum_i^n e_i^2$
- Equation of the line is: $\hat{y} = b_0 + b_1 x$
- **Slope** of the line is:b_1, where the slope measures the amount of change caused in the response variable when the explanatory variable is increased by one unit.
- **Intercept** of the line is:b_0, where the intercept is the value of the response variable when the explanatory variable = 0. (i.e. value where line intersects the y-axis)
- Used for Prediction: using the line to find y-values corresponding to x-values that are within the range of your data x-values
- Using values outside range of the collected data can lead to **extrapolation**
- Coefficient of Determination: Denoted by r^2, it gives the proportion of the variance of the response variable that is predicted by the explanatory variable. So when r^2 is high, close to 1 or 100%, you have explained most of the variability
- Residuals: the difference between the observed value of the response variable (y) and the predicted value ($\hat{y}$): residuals = observed y - predicted y, $e = y - \hat{y}$
- Relationship between Correlation and Coefficient of Determination: $|r_{x,y}| = \sqrt{r^2}$

EXAMPLE 2

We want to examine whether the amount of rainfall per year increases or decreases corn bushel output. A sample of 10 observations was taken, and the amount of rainfall (in inches) was measured, as was the subsequent growth of corn.

Amount of Rain	Bushels of Corn
3.03	80
3.47	84
4.21	90
4.44	95
4.95	97
5.11	102
5.63	105
6.34	112
6.56	115
6.82	115



The regression line (also called the prediction line or trend line) is $\hat{y} = 50.8350 + 9.6252x$ with $r^2 = 0.9906$ On the rain/corn data above, predict the corn yield for

1. 5 inches of rain
2. 6.56 inches of rain
3. 0 inches of rain
4. 100 inches of rain
5. For which amounts of rainfall above do you think the line does a good job of predicting actual corn yield? Why?
6. What percentages of the variation in corn yield is explained by the relationship with amount of rain?
7. Calculate the residual when the amount of rain is 6.56 inches.
8. Calculate the Correlation and compare it with the square root of r^2.