INTRODUCTION TO PROBABILITY MODELS

Lecture 30

Qi Wang, Department of Statistics

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THE EMPIRICAL RULE

$X \sim N(\mu, \sigma)$

- Approximately 68% of the observations fall within 1σ of the mean μ
  $P(\mu - \sigma < X < \mu + \sigma) = 0.68$
- Approximately 95% of the observations fall within 2σ of the mean μ
  $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.95$
- Approximately 99.7% of the observations fall within 3σ of the mean μ
  $P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.997$

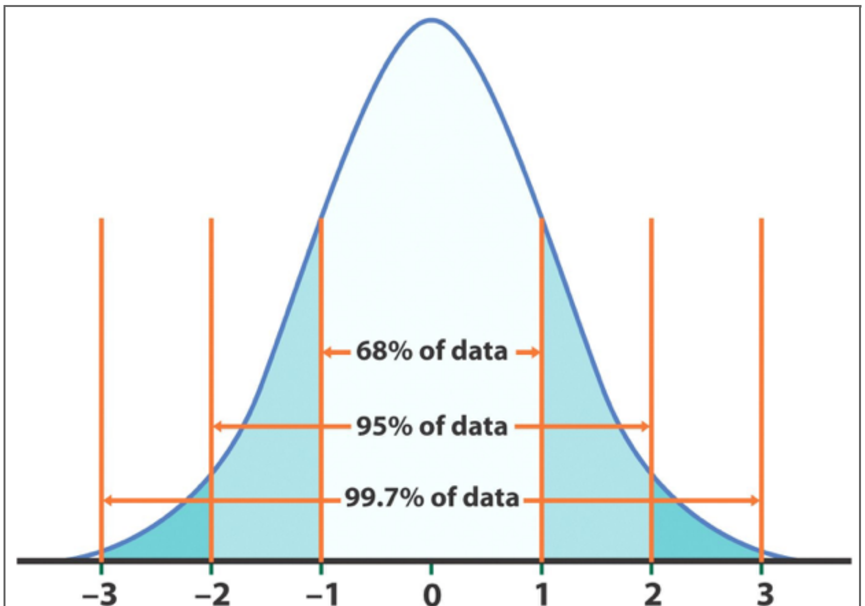

Figure 1-27
Introduction to the Practice of Statistics, Fifth Edition

EXAMPLE 1

Checking account balances, X, are approximately normal with a mean of 1500 dollars and a standard deviation of 50 dollars

1. Between what numbers do 95% of the balances fall?
2. Above what number do 2.5% of the balances lie?
3. Approximately what % of balances are between 1400 dollars and 1550 dollars?
4. Approximately what % of the balances are less than 1450 dollars?

“BACKWARDS” NORMAL PROBLEMS

What if we know the probability and want to find the related z-score and the value in the original distribution? We will work the table backwards!

Steps to finding the sample score if you are given a probability and know $X \sim N(\mu, \sigma)$

1. Set up your problem as follows $P(Z \leq z_0) = probability$ (Note: adjust $>$ to $\leq$ if necessary by using “1-probability”.)
2. Find the z-score by looking up the probability in the body of normal table
3. If you have a two-sided probability, use $P(-z_0 < Z \leq z_0) = 2P(Z \leq z_0) - 1 = 2\Phi(z_0) - 1$
4. Convert the z-score to x using $z = \dfrac{x - \mu}{\sigma}$

EXAMPLE 2

Find for each of the following:

1. $P(Z < z_0) = 0.5$
2. $P(Z < z_0) = 0.9846$
3. $P(Z < z_0) = 0.95$
4. $P(Z > z_0) = 0.512$
5. $P(-z_0 < Z < z_0) = 0.5$

EXAMPLE 3

If $X \sim N(\mu = 4, \sigma = 1.5)$, find x_0 for each of the following:

1. $P(X < x_0) = 0.95$
2. $P(X < x_0) = 0.90$
3. $P(X > x_0) = 0.9236$
4. Find 2 values between which the center 30% of the data lies.

EXAMPLE 4

The weekly amount spent for maintenance and repairs at a certain company has an approximately normal distribution with a mean of 650 dollars and a standard deviation of 35 dollars.

1. What is the probability that the company spends less than 675 dollars on maintenance and repairs in one week?
2. If 725 dollars is budgeted to cover the maintenance/repairs for next week, what is the probability that the actual cost will exceed the budgeted amount?
3. For planning purposes, the company wants to know the range for the middle 60% of the distribution of weekly maintenance and repair costs. Find the values that determine the middle 60% of the distribution of maintenance/repair costs.
4. What should the company expect their maintenance/repair costs to be for a year? .