INTRODUCTION TO PROBABILITY MODELS

Lecture 29

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Oct 31, 2018

NORMAL RANDOM VARIABLE

- **Support:** $X \in (-\infty, +\infty)$
- **Parameter:**
  - μ: the mean of the random variable, determines the center of the distribution
  - σ: the standard deviation of the random variable, determines the shape of the distribution
- **PDF:** $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
- **CDF:** $F_X(x) = P(X \leq x)$, no closed-form expression
- **Expected Value:** $E[X] = \mu$
- **Variance:** $Var(X) = \sigma^2$
- **Notation:** $X \sim Normal(\mu, \sigma)$ or $X \sim N(\mu, \sigma)$

STANDARD NORMAL DISTRIBUTION

So, what if you need different probabilities for $X \sim N(\mu, \sigma)$? We'd need an infinite number of tables or a way to calculate the probabilities for infinite combination of and values. One way is through calculus (i.e. integrate the pdf) OR We standardize (convert) the particular normal distribution and use ONE TABLE for all.

- The standard normal distribution is the normal distribution with $\mu = 0, \sigma = 1$, namely, $X \sim N(\mu = 0, \sigma = 1)$
- The CDF of standard normal distribution is denoted as $\Phi(x)$
- You convert $X \sim N(\mu, \sigma)$ to $Z \sim N(\mu = 0, \sigma = 1)$, where Z has the standard Normal distribution. Convert/standardize using:
  $$Z = \frac{X - \mu}{\sigma}$$
  This standardized value is called a Z-score
- z-scores tell you how many standard deviation the original observation falls from the mean. For example a z-score = 1.0, tells you that the particular value is exactly one standard deviation above the mean
- z-scores are what you need in order to use the Standard Normal Table
- z-scores also let you compare two values from different Normal distributions to see their probabilities on the same scale.

Z-SCORE

$P(Z \leq z - score)$ is what you will find on the Normal table. To use the table:

- z-scores run down the left-most column of the tables. The 2nd decimal place of the z-score runs across the top-most row of the tables.
- The inner numbers are the probability that you are at or lower than your z-score
- The top part of the table has negative z-scores, the second page has positive z-scores.
- Remember that your table gives you the probability $P(Z \leq z) = \Phi(z)$
- $P(Z > z) = 1 - \Phi(z)$
- $P(a < Z < b) = \Phi(b) - \Phi(a)$

EXAMPLE 1

Use the Normal table to find the following probabilities:

1. $P(Z < 1.48)$
2. $P(Z \leq 1.48)$
3. $P(Z > 1.48)$
4. $P(Z < -1.48)$
5. $P(Z = 1.48)$
6. $P(-1.48 < Z < 1.48)$
7. $P(Z < -4.9)$
8. $P(Z < 5.34)$

EXAMPLE 2

If $X \sim N(\mu = 4, \sigma = 1.5)$, use the Normal table to find the following probabilities:

1. $P(X < 3)$
2. $P(X > 4.5)$
3. $P(3 < X < 4.56)$
4. $P(X < 4)$
5. $P(X = 3.45)$
6. $P(X > 11)$