INTRODUCTION TO PROBABILITY MODELS

Lecture 25

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EXAMPLE 1

At a high school track and field tournament, Mark's high jumps vary evenly from 1.8 meters to 2.15 meters, while Dan's high jumps vary evenly from 1.75 to 2.3 meters.

1. Let M be the length of one of Mark's high jumps. What are the distribution and parameter(s) of M?
2. What is that the probability that Mark jumps between 1.88 and 2.05 meters?
3. Which jumper's jumps has the smaller standard deviation?
4. What is the probability that one of Dan's high jumps is exactly 2.0 meters?
5. What length cuts off the highest 25% of Dan's high jumps?

EXPONENTIAL RANDOM VARIABLE

- **The definition of X** : The waiting time until the first success
- **Support:** $X \in [0, +\infty)$ or $X \geq 0$
- **Parameter:** μ, the average amount of time for one success
- **PDF:** $f_X(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}$, for $x \geq 0$
- **CDF:**

  $$F_X(x) = P(X \leq x) = \begin{cases} 0, x < 0 \\ 1 - e^{-\frac{x}{\mu}}, x \geq 0 \end{cases}$$
- **Expected Value:** $E[X] = \mu$
- **Variance:** $Var(X) = \mu^2$
- **Notation:** $X \sim Exp(\mu)$ or $X \sim Exponential(\mu)$

IMPORTANT PROPERTIES FOR THE EXPONENTIAL DISTRIBUTION

If $X \sim Exp(\mu)$

- Tail Probability formula: $P(X > x) = e^{-\frac{x}{\mu}}$
- Memoryless Property:
  $P(X > s + t | X > s) = P(X > t)$

EXAMPLE 2

It is your birthday and you are waiting for someone to write you a birthday message on Facebook. On average (on your birthday) you receive a facebook message every 10 minutes. Assume that birthday messages arrive independently.

1. What is the probability the next posting takes 15 minutes or longer to appear? What distribution, parameter(s) and support are you using?
2. What is the standard deviation of the time between birthday postings?
3. What is the probability that it takes 12.5 minutes for the next birthday posting?
4. Suppose that the most recent birthday posting was done at 1:40 pm and it is now 1:45 pm. What is the probability that you will have to wait until 1:53 pm or later for the next message?
5. What is the probability that your wait time for the next three messages is less than 8 minutes?
6. What is your median waiting time for birthday messages?

EXPONENTIAL RANDOM VARIABLE

- **The definition of X** : The waiting time until the first success
- **Support:** $X \in [0, +\infty)$ or $X \geq 0$
- **Parameter:** λ, the number of success per time unit, $\lambda = \frac{1}{\mu}$
- **PDF:** $f_X(x) = \lambda e^{-\lambda x}$, for $x \geq 0$
- **CDF:**
  $$F_X(x) = P(X \leq x) = \begin{cases} 0, x < 0 \\ 1 - e^{-\lambda x}, x \geq 0 \end{cases}$$
- **Expected Value:** $E[X] = \frac{1}{\lambda}$
- **Variance:** $Var(X) = \frac{1}{\lambda^2}$
- **Notation:** $X \sim Exp(\lambda)$ or $X \sim Exponential(\lambda)$

EXAMPLE 3

You are teaching your new puppy to fetch a ball and are interested in the amount of time it takes for the puppy to run, get the ball and bring it back to you after you throw it.

- Scenario 1: The average amount of time it takes is 30 seconds.
- Scenario 2: The amount of time it takes is anywhere from 10 to 45 seconds.

1. What is the distribution, parameter(s) and support for Scenario 1 and Scenario 2
2. For each scenario, find the probability that it takes more than 22 seconds for the puppy to fetch the ball. Less than 50 seconds??
3. What is the probability that it will take the puppy less than 40 seconds to fetch the ball knowing that it took the puppy longer than 22 seconds
4. Assuming independence, what is the probability that it takes the puppy less than 40 seconds to fetch each of the next 5 balls?