INTRODUCTION TO PROBABILITY MODELS

Lecture 23

Qi Wang, Department of Statistics

Oct 17, 2018

LAST THREE DISCRETE RANDOM VARIABLES

- Poisson
- Geometric
- Negative Binomial

POISSON DISTRIBUTION

- $X \sim Poisson(\lambda)$
- **The definition of X**: the number of success per __, and __ can be time, length, space unit and so on
- **Support:** $\{0, 1, 2, \cdots\}$
- **Parameters:** λ, the average success rate per __
- **PMF:** $P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$
- **Expected Value:** $E[X] = \lambda$
- **Variance:** $Var(X) = \lambda$

GEOMETRIC DISTRIBUTION

- $X \sim Geom(p)$
- **The definition of X** : the number of trials to get the first success
- **Support:** $\{1, 2, \cdots\}$, NOTE: **NO ZERO!**
- **Parameter:** p, the probability of success in one trial
- **PMF:** $P_X(x) = p(1-p)^{x-1}$
- **Expected Value:** $E[X] = \frac{1}{p}$
- **Variance:** $Var(X) = \frac{1-p}{p^2}$
- Tail Probability formula: $P(X > k) = (1-p)^k$
- Memoryless Property:
  $P(X > s + t | X > s) = P(X > t)$ and
  $P(X < s + t | X > s) = P(X < t)$

NEGATIVE BINOMIAL DISTRIBUTION

- $X \sim NegBin(r, p)$ or $X \sim NB(r, p)$
- **The definition of X** : the number of trials to get the r_{th} success
- **Support:** $\{r, r+1, r+2, \cdots\}$
- **Parameter:**
  - p: the probability of success in one trial
  - r: success of interest
- **PMF:** $P_X(x) = C_{r-1}^{x-1} p^r (1-p)^{x-r}$
- **Expected Value:** $E[X] = \frac{r}{p}$
- **Variance:** $Var(X) = \frac{r(1-p)}{p^2}$

TIME FOR QUIZ

PROPERTIES OF EXPECTED VALUE AND VARIANCE

X, Y are random variables, c and d are constant

- $E[c] = c$
- $E[cX] = cE[X]$
- $E[X + Y] = E[X] + E[Y]$
- $E[cX + dY] = cE[X] + dE[Y]$
- $Var(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2$
- $Var(c) = 0$
- $Var(cX) = c^2\, Var(X)$
- If X and Y are independent,
  $Var(X + Y) = Var(X) + Var(Y)$
- If X and Y are independent,
  $Var(cX + dY) = c^2\, Var(X) + d^2\, Var(Y)$