

# INTRODUCTION TO PROBABILITY MODELS

Lecture 20

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# GEOMETRIC DISTRIBUTION REVISIT

- $X \sim \text{Geom}(p)$
- **The definition of  $X$**  : the number of trials to get the first success
- **Support:**  $\{1, 2, \dots\}$ , NOTE: **NO ZERO!**
- **Parameter:**  $p$ , the probability of success in one trial
- **PMF:**  $P_X(x) = p(1 - p)^{x-1}$
- **Expected Value:**  $E[X] = \frac{1}{p}$
- **Variance:**  $\text{Var}(X) = \frac{1-p}{p^2}$
- Two important properties:
  - Tail Probability formula:  $P(X > k) = (1 - p)^k$
  - Memoryless Property:  
 $P(X > s + t | X > s) = P(X > t)$  and  
 $P(X < s + t | X > s) = P(X < t)$

# NEGATIVE BINOMIAL RANDOM VARIABLE

## NEGATIVE BINOMIAL RANDOM VARIABLE

- **The definition of X** : the number of trials to get the  $r_{th}$  success
- **Support**:  $\{r, r + 1, r + 2, \dots\}$
- **Parameter**:
  - $p$ : the probability of success in one trial
  - $r$ : success of interest
- **PMF**:  $P_X(x) = C_{r-1}^{x-1} p^r (1 - p)^{x-r}$
- **Expected Value**:  $E[X] = \frac{r}{p}$
- **Variance**:  $Var(X) = \frac{r(1-p)}{p^2}$
- $X \sim NegBin(r, p)$  or  $X \sim NB(r, p)$

## EXAMPLE 1

Pat is required to sell candy bars to raise money for the 6<sup>th</sup> grade field trip. He will ask his neighbors to buy a candy bar. There is a 40% chance of him selling a candy bar to any neighbor that he asks. He has to sell 5 candy bars in all. (Note: anyone purchasing will only buy ONE candy bar and the neighbors are independent of each other).

1. What is the probability that he must ask 10 neighbors to sell all his candy bars?
2. What is the probability that he asks fewer than 9 neighbors?
3. How many neighbors does he expect to ask in order to sell all his candy bars?

## EXAMPLE 2

The Plattsville Pluggers are a minor league baseball team. Suppose that their ability to win any one game is 42% and games are independent of one another.

1. What is the probability that it takes 14 games to get their 4<sub>th</sub> win?
2. What is the expected value and standard deviation of the number of games to get their 4<sub>th</sub> win? Their 25<sub>th</sub> win? Their 1<sub>st</sub> win?
3. Knowing that the Pluggers got their 49<sub>th</sub> win with 5 games remaining in the season, what is the probability that they do NOT get 50 or more wins?

# RELATIONSHIP BETWEEN GEOMETRIC DISTRIBUTION AND NEGATIVE BINOMIAL DISTRIBUTION

- Geometric a special case of the Negative Binomial w/
- $X_i \sim \text{Geom}(p), i = 1, 2, \dots, r$ , then  
$$X = X_1 + X_2 + \dots + X_r \sim \text{NegBin}(r, p)$$
- $E[X] = E[X_1 + X_2 + \dots + X_r] = E[X_1] + E[X_2] + \dots + E[X_r]$
- $\text{Var}(X) = \text{Var}(X_1 + X_2 + \dots + X_r) = \text{Var}(X_1) + \dots + \text{Var}(X_r)$

### EXAMPLE 3

The wonderful candy shop, Albanese Candy Outlet, makes chocolate chip cookies as part of their production line. Chocolate chips in the cookies are randomly and independently distributed with an average of 12 chocolate chips per cookie. You and 9 of your friends decide to make a trip to Albanese Candy Outlet. Each of you buys one chocolate chip cookie.



1. What is the probability that your cookie contains between 10 and 15 chocolate chips inclusive?
2. What is the probability that 5 or 6 people in your group have cookies with between 10 and 15 chocolate chips inclusive?
3. While examining your cookies (one-by-one), what is the probability that it takes at least 4 cookies to find the first one with between 10 and 15 chocolate chips inclusive?
4. While examining your cookies (one-by-one), what is the probability that it takes at least 4 cookies to find the first one with 12 or 13 chocolate chips?
5. Suppose you and your 9 friends were to go repeatedly to Albanese Candy Outlet. What is the probability that it takes until your sixth trip so that 5 or 6 people in your group have 12 or 13 chocolate chips in their cookie?