INTRODUCTION TO PROBABILITY MODELS

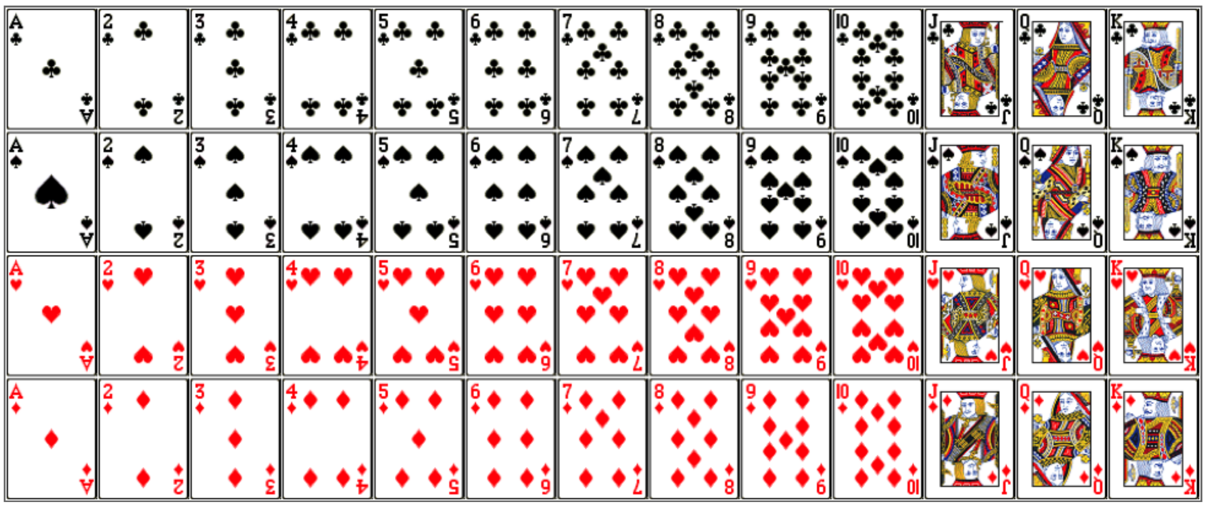
Lecture 2

Qi Wang, Department of Statistics

Aug 22, 2018

EXAMPLE 1

Consider a standard deck of 52 playing cards. You will draw one card out of a thoroughly shuffled deck of cards.



Define the following events: H = the suit is a heart, R = the card is red, F = the card is a face card (i.e. Jack, Queen or King)

1. Determine which of these events are subsets of each other?
2. Are any of these sets complements of each other? Describe what would be in H^c? In R^c?
3. What are the chances that you will draw out a spade? A seven? A red jack?

PROBABILITY

- Probability: the chance or likelihood of a random event occuring

We determine probablity one of two ways:

- Equally likely framework: know all possible outcomes and each outcomes is equally likely
  $$P(E) = \frac{\#OutcomesInEvent}{\#OutcomesInSampleSpace} = \frac{N(E)}{N(\Omega)}$$
- Frequentist interpretation of probability: probability is the long run proportion of times event occurs in independent repetitions of random experiment
  $$P(E) = \frac{N(E)}{n}$$
  n is sample size and n is large

Which would apply to these?

- Probability that a student is wearing blue jeans?
- Probability that in a roll of a die you will get a 5?
- Probability that it will snow in January?

PROBABILITY RULES

- Any probability must be between 0 and 1(inclusive), $0 \leq p \leq 1$
- Sum of probability of all outcomes in Ω must equal to 1, $P(\Omega) = 1$, $\sum_{i=1}^{n} p_i = 1$
- probability of event is the sum of probability of the specific outcomes in that event

Legitimate: If Rule 1 and Rule 2 are met, a probability model is legitimate

Question: Is $A \subset B$, what can you say about their probabilities?

EXAMPLE 2

Your friend tells you the following probabilities for a weighted die. You will roll the die one time.

Die Roll(X)	1	2	3	4	5	6
Probability	0.5	0.1	0.1	0.1	0.1	0.1

1. Is this a legitimate probability model?
2. What is the probability of rolling a 4 or higher?
3. What is the probability of rolling an even number?

EXAMPLE 3

A fair six-sided die is rolled twice

1. Write out the sample space using correct notation
2. Define the following events:
   - J = the two rolls are the same number;
   - K = the sum of the rolls is at least 4;
   - L = the sum of the rolls is 7.

   FIND $P(J)$; $P(K)$; $P(L)$
3. What is $P(K^c)$?

VENN DIAGRAMS

- **Venn Diagrams**: a diagram that shows all possible logical relations between a finite collection of different sets. They are useful tools for visualizing probability models
- **Intersection** of events A and B consists of those outcomes that are in **BOTH** event A and in event B, denoted by $A \cap B$, $\cap$ = intersection

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- **Union** of events A and B consists of those outcomes that in either event A or in event B or in BOTH, denoted by $A \cup B$, $\cup$ = union

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- NOTE: both intersections and unions can be determined for 3 or more events.

EXAMPLE 3 CONTINUED

- Find $P(J \cap K)$; $P(J \cap L)$; $P(K \cap L)$
- Find $P(J \cup K)$; $P(J \cup L)$; $P(K \cup L)$
- Find $P(J \cap L^c)$.

DEFINITIONS

- **Mutually Exclusive(disjoint):** $A \cap B = \emptyset$
- **Exhaustive:** $A \cup B = \Omega$
- **Partition:**
  - $A \cap B = \emptyset$
  - $A \cup B = \Omega$

MORE PROBABILITY LAW AND RULES

- $P(\Omega) = 1, P(\emptyset) = 0$
- Associative Laws:
  - $A \cap (B \cap C) = (A \cap B) \cap C$
  - $A \cup (B \cup C) = (A \cup B) \cup C$
- Commutative Laws:
  - $A \cap B = B \cap A$
  - $A \cup B = B \cup A$
- Distributive Laws:
  - $A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- Complement Rule: $P(A^c) = 1 - P(A)$
- General Additive Rule:
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$