INTRODUCTION TO PROBABILITY MODELS

Lecture 19

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NAMED RANDOM VARIABLES

- Bernoulli
- Binomial
- Hypergeometric
- Poisson
- ~~Geometric~~

BERNOULLI DISTRIBUTION

- $X \sim Bern(p)$
- **The definition of X**: the success of some event on a single trial.
- **Support:** $\{0, 1\}$
- **Parameter:** p
- **PMF:** $P_X(x) = p^x(1-p)^{1-x}$
- **Expected Value:** $E[X] = p$
- **Variance:** $Var(X) = p(1-p)$

BINOMIAL DISTRIBUTION

- $X \sim Binomial(n, p)$
- **The definition of X**: the total number of successes in a sequence of n independent Bernoulli experiments, with a success rate p
- **Support:** $\{0, 1, 2, \cdots, n\}$
- **Parameter:** n, p
- **PMF:** $P_X(x) = C_x^n p^x (1-p)^{n-x}$
- **Expected Value:** $E[X] = np$
- **Variance:** $Var(X) = np(1-p)$

HYPERGEOMETRIC DISTRIBUTION

- $X \sim Hyper(N, n, M)$
- **The definition of X**: the number of success in n trail without replacement from a finite population of size N that contains exactly M objects with that feature.
- **Support:** $\{0, 1, 2, \cdots, n\}$ or $\{0, 1, 2, \cdots, M\}$
- **Parameters:**
  - N : Population size
  - M : Number of possible successes
  - n : Number of trials
- **PMF:** $P_X(x) = \frac{C_x^M C_{n-x}^{N-M}}{C_n^N}$
- **Expected Value:** $E[X] = n\frac{M}{N}$
- **Variance:** $Var(X) = n\frac{M}{N}(1 - \frac{M}{N})\frac{N-n}{N-1}$

POISSON DISTRIBUTION

- $X \sim Poisson(\lambda)$
- **The definition of X**: the number of success per ____, and ____ can be time, length, space unit and so on
- **Support:** $\{0, 1, 2, \cdots\}$
- **Parameters:** λ, the average success rate per ____
- **PMF:** $P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$
- **Expected Value:** $E[X] = \lambda$
- **Variance:** $Var(X) = \lambda$

TIME FOR QUIZ

EXAMPLE 1

In a recent year, the Wall Street Journal, reported that 58% of all American credit card holders had to pay a late fee. A random sample of 15 American credit card holders is selected. Let X be the number of credit card holders in the sample who had to pay a late fee. Assume that all credit card holders are independent of one another.

1. State the distribution and parameter(s) for X. What is the support for X?
2. What is the average and standard deviation of X?
3. What is the probability that exactly 8 people in the sample had to pay a late fee?
4. Given that at least one person in the sample had to pay a late fee, what is probability that 8 or 9 had to pay a late fee?

EXAMPLE 2

A college student is running late for his class and does not have time to pack his backpack carefully. He has 12 folders on his desk, 4 include HW assignments due today. He grabs 3 of the folders randomly and when he gets to class, counts the number of them that contain HW.

1. What is the random variable here? What are the parameters?
2. What is the expected number of folders with HW in them? What is the variance of X?
3. What is the probability that 2 folders contain HW?
4. What is the probability that fewer than 2 folders contain HW?

EXAMPLE 3

Courtney is running downtown and passes a gas station at a rate of once per 2.5 minutes (this is an average of 0.4 gas stations a minute). Let G be the number of gas stations Courtney passes during her thirty minute run.

1. What is the support, distribution and parameter(s) of G?
2. What is the probability Courtney passes 10 gas stations on her run?
3. Courtney decides that to provide motivation, at the end of the run she will eat half a cookie for every gas station she passed on the run. What is the expected number of cookies Courtney will eat after the run?