

# INTRODUCTION TO PROBABILITY MODELS

Lecture 18

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## REMINDERS

1. The fourth quiz will be on this **Wednesday, Oct 3**

## EXAMPLE 1

Consider a game where we will roll a fair die. We will roll it until we get a 5. What is the probability that it will take 3 rolls to get the 5?

Think about:

- Are the subsequent rolls independent?
- What about the probability of success on each roll?
- Do we have a set number of trials?

# GEOMETRIC RANDOM VARIABLE

# CHARACTERISTICS OF THE GEOMETRIC DISTRIBUTION

- **The definition of X** : the number of trials to get the first success
- **Support:**  $\{1, 2, \dots\}$ , NOTE: **NO ZERO!**
- **Parameter:**  $p$ , the probability of success in one trial
- **PMF:**  $P_X(x) = p(1 - p)^{x-1}$
- **Expected Value:**  $E[X] = \frac{1}{p}$
- **Variance:**  $Var(X) = \frac{1-p}{p^2}$
- $X \sim Geom(p)$

## EXAMPLE 2

Suppose Dunphy is really bad at tossing a Frisbee and unfortunate hits people walking by at a rate of 1 out of every 5 people.

1. What is the probability that his first accidental hitting is the 6<sup>th</sup> or 7<sup>th</sup> person to walk by?
2. What is the probability that more than 7 people walk past before he hits one with the Frisbee?

## IMPORTANT PROPERTIES FOR THE GEOMETRIC DISTRIBUTION

- Tail Probability formula:  $P(X > k) = (1 - p)^k$
- Memoryless Property:  
 $P(X > s + t | X > s) = P(X > t)$  and  
 $P(X < s + t | X > s) = P(X < t)$

## EXAMPLE 2 CONTINUED

- 3. Four people have walked past Dunphy and none have been hit by a Frisbee. What is the probability that at most 9 walk by until the first person is hit by a Frisbee?
- 4. Four people have walked past Dunphy and none have been hit by a Frisbee. What is the probability that at least 10 walk by until the first person is hit by a Frisbee?



### EXAMPLE 3

Shaq is shooting free throws in the gym. He intends to stay until he makes one. His probability of making one on any free throw is 0.527. Let  $X$  be the number of attempts until he makes one.

1. Distribution, parameter, support?
2. Expected number of shots until he makes one?
3. Probability he makes his first shot on the 4<sub>th</sub> try?
4. Probability it takes him at least 4 shots to make 1<sub>st</sub>?
5. Probability it takes him exactly 4 shots if he already missed the first?
6. Probability it takes him at least 4 shots if he already missed the first?

## EXAMPLE 4

Bugcatcher Nathan is standing by the side of the road with his Caterpie, ready to start battling other trainers with his Pokémon. Since he's just starting out, Nathan has a very low chance of winning each battle, just 16%. Let  $X$  be the number of battles Nathan must endure with his Caterpie before he finally wins his first.

1. What is the support, distribution, and parameter(s) of  $X$
2. Nathan makes 100 Pokédollars for every win, but has to pay 20 Pokédollars for each loss. Luckily, Nathan has deep pockets. How much money would you expect Nathan to have won, or lost, in total after his first win?
3. If Nathan has battled 7 trainers so far today, what is the probability that he faces at least 10 total before he earns his first win?