INTRODUCTION TO PROBABILITY MODELS

Lecture 17

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POISSON DISTRIBUTION

POISSON DISTRIBUTION

- **The definition of X**: the number of success per __, and __ can be time, length, space unit and so on
- **Support:** $\{0, 1, 2, \cdots\}$
- **Parameters:** λ, the average success rate per __
- **PMF:** $P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$
- **Expected Value:** $E[X] = \lambda$
- **Variance:** $Var(X) = \lambda$
- $X \sim Poisson(\lambda)$

RULE FOR USING A POISSON TO APPROXIMATE A BINOMIAL RANDOM VARIABLE:

If $X \sim Bin(n, p)$ with **n $>$ 100 AND p $<$ 0.01**, we can approximate X by $X^{\star} \sim Poisson(\lambda = np)$

EXAMPLE 1

Suppose earthquakes occur in the western US on average at a rate of 2 per week. Let X be the number of earthquakes in the western US this week.

- Find the probability that X is 3. What distribution and parameter(s) are you using?
- What is the probability that there are at least 2 earthquakes in a week in the western US?
- What is the expected number of earthquakes and the standard deviation of the number of earthquakes in the western US in a week?

Now consider a month. Let Y be the number of earthquakes in the western US this month(assume that 1 month is equivalent to 4 weeks).

- Find the probability that Y is 12. What distribution and parameter(s) are you using?
- Let Z be the number of weeks in a 4 week period that have a week with 3 earthquakes in the western US. Find the probability that Z is 4. Is this the same as the probability that Y is 12? Does this make sense?

EXAMPLE 2

Customers arrive at the UPS store randomly and independently at a rate of 15 per hour.

1. What is the probability that 45 customers arrive between 11:30 am and 3:00 pm? What distribution and parameter(s) are you using? What is the support?
2. What is the probability that 45 customers arrive between 11:30 am and 3:00 pm AND 10 customers arrive between 3:00 pm and 3:45 pm?

EXAMPLE 3

Flaws on an old computer tape occur on average every 1200 feet. You have an old computer tape roll that is 4800 feet long.

1. What is the probability that there is at least one flaw on that roll?
2. You know that there is at least one flaw on the roll. Knowing this, what is the probability that there are 2 or 3 flaws on the roll?

EXAMPLE 4

A certain disease occurs in 7 out of 5000 people. We will conduct a study and take a sample of 1000 people. What is the probability that no one in the sample has the disease?