INTRODUCTION TO PROBABILITY MODELS

Lecture 15

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REMINDERS

- The first exam will be at **CL50**(Class of 1950 Lecture Hall) from **8:00pm to 9:30pm on Tuesday, Sep 25**

CONCEPTS

- Random Experiment:
- Element
- **Event**
  - $\emptyset$
  - S or Ω
- **Union** $\cup$:
- **Intersection** $\cap$:
- **DeMorgan's Law:**
- **Mutually Exclusive, Exhaustive and Partition**

CONCEPTS

- **Conditional Probability:** if event B has nonzero probability ($P(B) > 0$), $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- **Multiplication Rule:** $P(A \cap B) = P(B) \times P(A|B)$
- **General addition rule:**
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - $P(A \cup B \cup C)$
- **Independence:**
  - $P(A|B) = P(A)$
  - $P(B|A) = P(B)$
  - $P(A \cap B) = P(A) \times P(B)$
  - Mutually exclusive events are NOT independent unless one of them has zero probability
  - Mutually Independent and Pairwise Independent
- **Law of Total Probability:** $P(A) = \sum_{i=1}^{n} P(A|B_i) \times P(B_i)$
- **Bayes Rule:** $P(B_i|A) = \frac{P(B_i \cap A)}{\sum_{i=1}^{n} P(A|B_i) \times P(B_i)}$

CONCEPTS

- **Basic Counting Rules**
- **Permutation:** ${}_nP_r = P_r^n = \frac{n!}{(n-r)!}$
- **Combination:** ${}_nC_r = C_r^n = \frac{n!}{(n-r)!r!}$
- **Multinomial Coefficient:**
  $\binom{m}{m_1, m_2, \cdots, m_k} = \frac{m!}{m_1!m_2!\cdots m_k!}$
- Random Variable
- Probability Mass Function
  1. For every x, $0 \leq p_X(x) \leq 1$
  2. $\sum_x p_X(x) = 1$
- **Expected Value:** $E[X] = \sum_x x \times p_X(x)$
  - c is a constant, $E[cX] = cE[X]$
  - $E[X + Y] = E[X] + E[Y]$
- **Variance:**
  $Var(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2$
  - $Var(cX) = c^2\, Var(X)$
  - If X and Y are independent,
    $Var(X + Y) = Var(X) + Var(Y)$

TWO DIAGRAMS

1. Venn Diagram
2. Tree Diagram

EXAMPLES

- **Problem 10 in Sample Exam 1**
- **Problem 11 in Sample Exam 1**
- **Problem 23 in Exam 1 Study Guide**