INTRODUCTION TO PROBABILITY MODELS

Lecture 14

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REMINDERS

1. The second homework is due **NOW**

NAMED RANDOM VARIABLES

- Bernoulli
- Binomial
- Hypergeometric
- Poisson
- Geometric
- Negative Binomial

BERNOULLI DISTRIBUTION

- The probability distribution of a random variable which takes the value 1 with probability p and the value 0 with probability $q = 1 - p$
- The outcome is **YES or NO**, **SUCCESS or FAILURE**, **1 or 0** ⋯

BERNOULLI DISTRIBUTION

- $X \sim Bern(p)$
- **Support:** $\{0, 1\}$
- **Parameter:** p
- **PMF:** $P_X(x) = p^x(1-p)^{1-x}$
- **Expected Value:** p
- **Variance:** $p(1-p)$

EXAMPLE 1

In Eric's STAT 225 class, 80% of the students passed on Exam 1. If we were to pick a student at random and asked them whether or not they passed, let X be the number of students who passed. What type of random variable is this? How do you know? Additionally, write down the pmf, the expected value, and the variance of X.

BINOMIAL DISTRIBUTION

- $X \sim Binomial(n, p)$
- The total number of successes in a sequence of n independent Bernoulli experiments, with a success rate p
- **Support:** $\{0, 1, 2, \cdots, n\}$
- **Expected Value:** np
- **Variance:** $np(1 - p)$

EXAMPLE 2

Now pick 10 students from Eric's class, with the same probability of having passed. Let X be the total number of students who passed. What type of random variable is this? What values can X take? Please write down the pmf, the expected value, and the variance of X.

RELATIONSHIP BETWEEN BERNOULLI DISTRIBUTION AND BINOMIAL DISTRIBUTION

Theorem: Let $X_1, X_2, \cdots, X_n$ be independent Bernoulli random variables, each with the same parameter p. Then the sum $X = X_1 + X_2 + \cdots + X_n$ is a binomial random variable with parameters n and p