INTRODUCTION TO PROBABILITY MODELS

Lecture 13

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REVISION

- **Permutation:** Ordered arrangement of r distinct objects from a set of n objects.

  $${}_nP_r = P_r^n = \frac{n!}{(n-r)!}$$

- **Combination:**Unordered arrangement of r distinct objects from a set of n objects.

  $${}_nC_r = C_r^n = \frac{n!}{(n-r)!r!}$$

- **Multinomial Coefficient:**m objects are in k distinct groups, size of groups are $m_1, m_2, \cdots, m_k$, number of ways to order these are:

  $$\binom{m}{m_1, m_2, \cdots, m_k} = \frac{m!}{m_1!m_2! \cdots m_k!}$$

TIME FOR QUIZ

PROPERTIES OF EXPECTED VALUE

X, Y are random variables, c and d are constant

- $E[c] = c$
- $E[cX] = cE[X]$
- $E[X + Y] = E[X] + E[Y]$
- $E[cX + dY] = cE[X] + dE[Y]$

PROPERTIES OF VARIANCE

X, Y are random variables, c and d are constant

- $Var(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2$
- $Var(c) = 0$
- $Var(cX) = c^2\, Var(X)$
- If X and Y are independent,
  $Var(X + Y) = Var(X) + Var(Y)$
- If X and Y are independent,
  $Var(cX + dY) = c^2\, Var(X) + d^2\, Var(Y)$

EXAMPLE 1

Suppose X and Y are random variables with $E[X] = 3, E[Y] = 4$ and $Var(X) = 2, Var(Y) = 1$. Find

1. $E[2X + 1]$
2. $E[X - Y]$
3. $E[X^2]$
4. $E[X^2 - 4]$
5. $E[(X-4)^2]$
6. $Var(2X - 4Y)$