INTRODUCTION TO PROBABILITY MODELS

Lecture 12

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Sep 17, 2018

EXPECTATION

EXPECTED VALUE OF A DISCRETE RANDOM VARIABLE

- **Definition:** Weighted average of the possible values,
  $E[X] = \sum_{x} x \times p_X(x)$
- Expected value can be positive or negative
- It does **NOT** have to be an integer

SOME PROPERTIES OF EXPECTED VALUE

- c is a constant, $E[cX] = cE[X]$
- $E[X + Y] = E[X] + E[Y]$

EXAMPLE 1

$X \sim p_X(x) = P(X = x) = k(5 - x), x \in \{0, 1, 2, 3, 4\}$, if X has the valid pmf, find the expected value of X

VARIANCE

VARIANCE OF A DISCRETE RANDOM VARIABLE

- **Definition:**measures of spread, relates how far a particular value of the r.v. is from the average (i.e. expected value) of the r.v
  $Var(X) = E[(X - E[X])^2] = E[X^2] - (E[X])^2$
- The variance will NEVER be negative.
- **Standard Deviation:** Square root of the variance
  $SD(X) = \sqrt{Var(X)}$

SOME PROPERTIES OF VARIANCE

- c is a constant, $Var(cX) = c^2 Var(X)$
- If X and Y are independent,
  $Var(X + Y) = Var(X) + Var(Y)$

EXAMPLE 2

For the unfair coin problem in Lecture 11, find
$E[3X - 2]$, $SD[3X - 2]$

EXAMPLE 3

Suppose X and Y are random variables with $E[X] = 3, E[Y] = 4$ and $Var(X) = 2$. Find

1. $E[2X + 1]$
2. $E[X - Y]$
3. $E[X^2]$
4. $E[X^2 - 4]$
5. $E[(X - 4)^2]$
6. $Var(2x - 4)$